

# Assignment Problem and Vehicle Routing Problem for an Improvement of Cash Distribution

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**Abstract**— We consider a case study on the application of techniques for solving assignment problem (AP) and vehicle problem with time window (VRPTW) occurred in cash distribution of bank in Bangkok, Thailand. An intensive review of the literature about AP and VRP is also discussed. The main aims of this research are to cluster all branches into groups belongs to each depot and to produce the routes for each depot. The objective of this research is to improve a cash distribution while using the existing resources. In order to find good solutions, an optimization of assignment problem is used incorporated with heuristics methods for VRP. Sweep Algorithm, Group Sweep Algorithm, and Nearest Neighbor Algorithm are used in this research. Results received by those methods are better than the current operation.

**Index Terms**— Vehicle Routing Problem, Sweep Method, Assignment Problem, Nearest Neighbor Algorithm, Group Sweep Algorithm.

## I. INTRODUCTION

**B**ANKing business is the financial institute that plays an important role in the economic system. In the past, major duty of bank is to circulate money in the economic system by raising money from saving and make it available for personal or business loan. Over the past few years, the overseas banks have been allowed to do business in Thailand. Since then, banking business has become a highly competitive business and Thai banks have developed "universal banking" concept in order to encounter or compete with the new comer. The meaning of "universal banking" is everything about money which aims to prepare readiness of service in all area of customer's needs includes saving, lending, paying, and insuring. The strategy of which allow the successful of the universal banking concept is the accessibility to all types of services. Online banking is one example of how to increase the accessibility. The limitation of online banking is that it allows the accessibility for only

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the customer who can access the internet system. Setting up new service points is another example which allows the accessibility to all customers. However, increasing in number of service points without increasing service vehicles causes late in delivery and pickup. This problem causes the increasing of overtime cost and reducing of customer satisfaction. Therefore, every bank tries to increase its transportation service in order to increase its customer satisfaction.

## II. CASE STUDY OF N&C BANK'S TRANSPORTATION SERVICE

N&C bank is one of the biggest banks in Thailand. It has total of 377 branches in Bangkok and has 3 distribution centers. Currently, its transportation routes are initiated by the experienced drivers. There are total of 29 routes using total of 27 vehicles which have the same capacity. Two vehicles serve two trips per day with overtime. The following is the list of assumptions that shaped our formulation of the model to solve N&C's problem:

- Each route will start from and end at the depot which it belongs to.
- The route in the morning and the evening will be the same.
- Travel times between each branch are known and accurate.
- Demands (i.e., amount of cash) at each of the branch are known.
- Processing time per branch (stop) is constant for every branch.

The constraints in this problem are:

- The capacity of each vehicle is strictly enforced by the insurance value of 200 million baht per trip.
- Hours of operation of each branch depends on where are it located. If it is located in the department store, its working hours are from 11 am until 8 pm. If it is located elsewhere, the operating hour is from 8.30 am to 3.30 pm.
- Each depot or each distribution center (DC) – A, B, and C – has different processing capacity at the proportion of 50 : 30 : 20.
- Each DC's operating time is from 8.00 am to 5 pm.

The ultimate goal of N&C on this research is to increase its transportation services while using its existing resources. Since N&C has 3 DCs which responsible in picking up and

delivery cash to and from each branch everyday in a given different time window. The most suitable methodology for this problem would be multiple depot vehicle routing problems with time window. However, the answer of this research is new routes which can handle changes in daily operations, i.e., change in demands, change in operation time, etc., therefore, N&C needs a methodology which can give results in a short processing time. The researcher decides to use easy and quick algorithm to solve this problem. There are two main methodologies used in this research. First, assignment problem is employed and capacitated vehicle routing problem with time window (VRPTW) is used later. The assignment problem clusters 377 branches into 3 groups each group belongs to each DC and VRPTW produces routes for each DC daily.

The assignment problem may be defined mathematically as follows. Let the distance between branch  $i^{th}$  and DC  $j^{th}$  is  $c_{ij}$  and *Capacity* is a parameter represents each DC's capacity. The objective function of the assignment algorithm is to minimize total distance between branches and DCs which they belong to while not exceed their processing capacity. The assignment formulation of this problem is as follows:

$$Min.Z = \sum_{j=1}^n \sum_{i=1}^m c_{ij}x_{ij} \quad (1)$$

$$\sum_{j=1}^n x_{ij} \leq Capacity_i, \forall i \quad (2)$$

$$\sum_{i=1}^m x_{ij} = 1, \forall j \quad (3)$$

$$all x_{ij} = binary \quad (4)$$

Constraint (2) ensures that number of branches to be responsible by each DC does not exceed the capacity of each DC, while one branch can belongs to only one DC as shown in (3). Constraint (4) states that variable  $x_{ij}$  is a binary variable that takes a value of 1 if branch  $i$  belongs to DC  $j$  and takes a value of 0, otherwise.

The VRPTW which used to route the routing of each depot may be defined mathematically as follows. Let  $G = (V, A)$  be a network where  $V = \{0, 1, \dots, n\}$  is the vertex set and  $A \subseteq V \times V$  is the arc set. Vertex 0 is the depot or DC and  $V \setminus \{0\}$  is the set of branch locations on the road network. Associated with vertex  $i \in V \setminus \{0\}$  is a non-negative demand  $d_i$ . The parameter  $c_{ij}$  represents a non-negative distance between vertices  $i$  and  $j$ . The parameters  $K$  and  $U_k$  are the number of vehicles and the capacity of vehicle  $k$ , respectively. [1] describes the VRP with an integer programming formulation using a three-index vehicle flow formulation where binary variables  $x_{ijk}$  count the number of times arc  $(i,j) \in A$  is traversed by vehicle  $k$  ( $k = 1, \dots, K$ ) in the optimal solution. In addition, there are binary variables  $y_{ik}$  ( $i \in V; k = 1, \dots, K$ ) that take a value of 1 if vertex  $i$  is visited by vehicle  $k$  in the optimal solution and take a value of 0, otherwise. The formulation is as follows:

$$\min \sum_{(i,j) \in A} c_{ij} \sum_{k=1}^K x_{ijk} + 0.33 \sum_{i \in V} Y_{ik} \quad (5)$$

subject to

$$\sum_{k=1}^K y_{ik} = 1 \quad \forall i \in V \setminus \{0\} \quad (6)$$

$$\sum_{k=1}^K y_{0k} = K \quad (7)$$

$$\sum_{j \in V \setminus \{i\}} x_{ijk} = y_{ik} \quad \forall i \in V, k = 1, \dots, K \quad (8)$$

$$\sum_{j \in V \setminus \{i\}} x_{jik} = y_{ik} \quad \forall i \in V, k = 1, \dots, K \quad (9)$$

$$\sum_{i \in V \setminus \{0\}} d_i y_{ik} \leq U_k \quad \forall k = 1, \dots, K \quad (10)$$

$$\sum_{(i,j) \in A} c_{ij} \sum_{k=1}^K x_{ijk} + 0.33 \sum_{i \in V} Y_{ik} \leq t \quad k = 1, \dots, K \quad (11)$$

$$\sum_{i \in S} \sum_{j \in S \setminus \{i\}} x_{ijk} \leq |S| - 1 \quad \forall S \subseteq V \setminus \{0\}, |S| \geq 2, k = 1, \dots, K \quad (12)$$

$$y_{ik} \in \{0,1\} \quad \forall i \in V, k = 1, \dots, K \quad (13)$$

$$x_{ijk} \in \{0,1\} \quad \forall (i, j) \in A, k = 1, \dots, K \quad (14)$$

Equation (5) represents the objective function of this problem to minimize total travel time of the operations. Constraints (6) - (9) ensure that each customer is visited exactly once, that  $K$  vehicles leave the depot, and that the same vehicle enters and leaves a given customer vertex, respectively. Constraints (10) are the capacity restrictions for each vehicle  $k$ , whereas constraint (11) is a time window constraint with a operation time of 20 minutes at each stop. The sub-tour elimination constraint for each vehicle is shown in constraint (12).

### III. LITERATURE REVIEW

In this section, we briefly review the literature in general of Assignment Problem (AP) and Vehicle Routing Problem (VRP) in various aspects, such as some variants of VRP and methodologies for VRP.

#### A. Assignment Problem

Assignment Problem is a method for matching the "Tasks" (jobs) to the "Agent" (man, machine of facility) which can produce the most efficient outcome. It is firstly introduced in 1952 [2]. AP can be categorized in to three groups – AP model with at most one task per agent, AP model with multiple tasks per agent, AP model for multi-dimensional assignment problem. An intensive review of

those techniques can be found in [3]. Ref [4] – [9] give a good of example of those technique applied to many situation.

### B. Vehicle Routing Problem

The most general version of the VRP is the Capacitated Vehicle Routing Problem (CVRP) which is a problem in which all customers must be satisfied, all demands are known, and all vehicles have identical, limited capacity and are based at a central depot. The objectives are to minimize the vehicle fleet and the sum of travel time while the total demand of commodities for each route may not exceed the capacity of the vehicle which serves that route [10], [11].

One of the most important extensions of the CVRP is the Vehicle Routing Problem with Time Window (VRPTW) which is each customer must be served within a specific time window. The objective is to minimize the vehicle fleet with the sum of travel time and waiting time needed to supply all customers in their required hour [10], [12]. A variety of exact algorithms and efficient heuristics have already been proposed for VRPTW by many researchers as shown in Table 1. In addition, Table 2 represents the various methods applying in exact algorithm, classical heuristic algorithms and metaheuristic algorithms for various type of VRP.

Table 1: Review of Vehicle Routing Problem with Time Window

Authors	Year	Methodologies
Dumas et al	1995	Time constraint routing and scheduling [13]
Liu and Shen	1999	Route-neighborhood-based metaheuristic [14]
Bent et al.	2003	Two-stage hybrid algorithm [15]
Kim, et al.	2005	Capacitated clustering [16]
Lysgaard	2006	Precedence constraints [17]
Russell and Chiang	2006	Robust solution methods [18]
Chena, Hsueh and Chang	2009	An elaborated solution Algorithms [19]
Li, et al.	2009	Lagrangian heuristic [20]

Table 2: Representative Exact algorithms, Classical Heuristic Algorithms and Metaheuristic Algorithms for VRP

a) Exact Algorithms		
Authors	Year	Methodologies
Christofides and Eilon	1969	Branch and Bound [21]
G. Laporte et al	1988	Branch and Bound [22]
Martin Desrochers et	1992	Column Generation [23]
Miller	1995	Branch and Bound [24]
Hadjiconstantinou et al	1995	Set-partitioning [25]
Ralph, T.K.	2003	Parallel Branch and Cut [26]
Baldacci et al.	2004	Branch and Cut [27]
Jin et al.	2008	Column Generation [28]

  

b) Classical Heuristic Algorithms		
Authors	Year	Methodologies
Dantzig and Ramser	1959	Constructive algorithm. First approach [29]
Clarke and Wright	1964	Saving. Concurrent & sequential [30]
Wren and Holliday	1972	Sweep Algorithm. Multiple depots [31]
Lin and Kernighan	1973	Single-route improvement Sequential k-exchange [32]

Gillett and Miller	1974	Sweep Algorithm. Single depot [33]
Foster and Ryan	1976	Petal algorithm. Optimal petal solution [34]
Mole and Jameson	1976	Sequential Route-Building Insertion position check [35]
Christofides et al.	1979	Sequential Route-Building Sequential & Parallel construction [36]
Fisher and Jaikumar	1981	Cluster-First Route-Second Generalized Assignment + TSP [37]
Beasley	1983	Route-First Cluster-Second [38]
Altinkemer and Gavish	1991	Matching Algorithm. Matching clusters [39]
Ryan et al.	1993	Petal algorithm [40]
Thompson and Psaraftis	1993	Multiple-Route Improvement b-cyclic k-transfer [41]
Potvin and Rousseau	1995	Single-route improvement. Based on 2-opt [42]
Bramel and Simchi-Levi	1995	Cluster-First Route-Second [43]
Renaud et al.	1996	Single-route improvement [44]
Kindervater and Savelsbergh	1997	Multiple-Route Improvement [45]

### c) Metaheuristic Algorithms

Authors	Year	Methodologies
Osman	1993	SA [46]
Taillard	1993	TS [47]
Gendreau et al.	1994	TS [48]
Van Breedam	1995	SA[49]
Rochat and Taillard	1995	TS [50]
Xu and Kelly	1996	TS [51]
Kawamura et al.	1998	ACO [52]
Bullnheimer et al.	1999	ACO [53]
Toth and Vigo	2003	TS [54]
Baker and Ayechev	2003	GA [55]
Mazzeo and Loiseau	2004	ACO [56]

## IV. A DECOMPOSITION METHODOLOGY TO SOLVE N&C'S PROBLEM

As stated in the previous section, we use two methodologies to solve N&C's problem. The assignment problem is firstly used to cluster all branches into groups. Each group belongs to each DC. The deterministic model for assignment problem which (1) – (4) can produce a good result within a short processing time. The result receives by AMPL with CPLEX 11.2 show that 152, 135, and 90 branches will be responsible by DC A, B, and C respectively.

Then, the routing technique is required. However, the conventional optimization VRP model can't produce a result within a small amount of time due to the model itself produces an exponentially growth associated with number of vertexes need to be served [57]. Hence, we decide to use the classical heuristics method for VRP to solve this problem. Sweep algorithm which is introduce since 1972 [31] is selected. The result received from this method is 28 routes without overtime.

A modified sweep is our second choice. This method starts from making groups of three closet branches and considering each group as one node. Then, an old fashion sweep algorithm starts. The result receives from group sweep is better than the current operation of the bank, because it produces total of 26 routes without overtime.

The third methodology used in this research is Nearest Neighbor Algorithm which can be considered as a Greedy

Method. This tool searches for the nearest branch to the depot as the starting point and then search for another branch connecting to the starting point using the same criteria which is closest branch. This step is repeat until the vehicle has maximum capacity. Then the new route starts again until all branches are served. The result received from this method is 23 routes.

Table 2: the result comparison in number of routes

D.C. Name	Nearest Neighbor	Sweep	Group Sweep	Today
A	9	11	10	11
B	8	10	9	11
C	6	7	7	7
Total	23	28	26	29

Table 2 shows a comparison of result received from all methods used for VRP. Even all methods give the better results in number of routes, however, the sweep method needs one more vehicle than N&C has. Currently, N&C decides to adopt Group sweep method, even the Nearest Neighbor Method is given the better result. This is because the current operation is easily to change. The result received from Nearest Neighbor Search will be used later.

#### V. SUMMARY AND RECOMMENDATION

From the research, we found that there are many literatures about AP and VPR. A simple AP model can produce an optimal cluster of each depot. Three easy methodologies are selected to solve VRP of N&C. Every algorithms produce good results within a short processing time. The authors have seen that Group Sweep Algorithm has potential to improve and will be our future research.

#### References

[1] Paolo Toth and Danniele Vigo, "The Vehicle Routing Problem", SIAM Monographs on Discrete Mathematics and Applications, 2002.  
[2] D.F. Votaw, A. Orden, The personnel assignment problem, Symposium on Linear Inequalities and Programming, SCOOP 10, US Air Force, 1952, pp. 155-163.  
[3] David W. Pentico, "Assignment problems: A golden anniversary survey", European Journal of Operational Research 176 (2007) 774-793.  
[4] G. Caron, P. Hansen, B. Jaumard, "The assignment problem with seniority and job priority constraints", Operations Research 47 (3) (1999) 449-454.  
[5] M. Dell'Amico, S. Martello, "The k-cardinality assignment problem", Discrete Applied Mathematics 76 (1-3) (1997) 103-121.  
[6] J. Kennington, Z. Wang, "A shortest augmenting path algorithm for the semi-assignment problem", Operations Research 40 (1) (1992) 178-187.  
[7] A. Volgenant, "Linear and semi-assignment problems: A core oriented approach", Computers & Operations Research 23 (10) (1996) 917-932.  
[8] S. Aora, M.C. Puri, "A variant of time minimizing assignment problem", European Journal of Operational Research 110 (2) (1998) 314-325.  
[9] K.C. Gilbert, R.B. Hofstra, "Multidimensional assignment problems", Decision Sciences 19 (2) (1988) 306-321.  
[10] Avalibale: [http://neo.lcc.uma.es/redi-ueb/WebVRP/Problem\\_Descriptions/VRPPDDesc.html](http://neo.lcc.uma.es/redi-ueb/WebVRP/Problem_Descriptions/VRPPDDesc.html) 10/18/2008  
[11] A. Boonkleaw, S. Suthikannarunai, R. Srinon, "Strategic Planning and Vehicle Routing Algorithm for Newspaper Delivery Problem: Case Study of Morning Newspaper, Bangkok, Thailand, Proceeding of the World Congress on Engineering and Computer Science, Sanfrancisco, USA, Vol. 2, 2009.  
[12] P. Mochado, et all, "Vehicle Routing Problem: Doing the Evolutionary Way"

[13] Desrosiers, J., Y. Dumas, M.M. Solomon and F. Soumis (1995), "Time Constrained Routing and Scheduling", In *Handbooks in Operations Research and Management Science, Vol. 8. Network Routing*, M.O. Ball, T.L. Magnanti, C.L. Monma, G.L. Nemhauser (eds), North-Holland, Amsterdam, pp. 35-139.  
[14] Fuh-Hwa Franklin Liu \*, Sheng-Yuan Shen "A route-Neighborhood-Based Metaheuristic for Vehicle Routing Problem With Time Windows, European Journal of Operational Research 118 (1999) 485-504.  
[15] R. Bent, P.V. Hentenryck, "A Two-Stage Hybrid Algorithm For Pickup and Delivery Vehicle Routing Problems With Time Windows", Computers & Operations Research 33 (2006) 875-893.  
[16] B. Kima., S. Kimb, S. Sahoob, "Waste Collection Vehicle Routing Problem With Time wWindows, Computers & Operations Research 33 (2006), 3624-3642.  
[17] Jens Lysgaard, "Reachability Cuts for the Vehicle Routing Problem with Time Windows", European Journal of Operational Research 175 (2006) 210-223.  
[18] Robert A. Russell, Wen-Chyuan Chiang, "Scatter Search For The Vehicle Routing Problem With Time Windows" European Journal of Operational Research 169 (2006) 606-622.  
[19] Huey-Kuo Chen, Che-Fu Hsueh., Mei-Shiang Chang, "Production Scheduling and Vehicle Routing with Time Windows for Perishable Food Products", Computers & Operations Research 36 (2009) 2311 - 2319.  
[20] Jing-Quan Li a,\*, Pitu B. Mirchandani, Denis Borenstein, Real-time Vehicle Rerouting Problems With Time Windows, European Journal of Operational Research 194 (2009) 711-727.  
[21] Christofides, N. and S. Eilon (1969, "September). An algorithm for the vehicle-dispatching problem". *Operational Research Quarterly* 20(3), 1969, pp.309-318.  
[22] G. Laporte, Y. Nobert, S. Taillefer, Solving a family of multi-depot vehicle routing and location-routing problems, Transportation Science 22 (1988) 161-172.  
[23] Martin Desrochers, Jacques Desrosiers, Marius Solomon, A New Optimization Algorithm for the Vehicle Routing Problem with Time Windows, *Operations Research*, Vol. 40, No. 2, 1992.  
[24] Miller, D. L., "A matching based exact algorithm for capacities vehicle routing problems". *ORSA Journal on Computing* 7(1), 1995, pp.1-9.  
[25] Hadjiconstantinou, E., N. Christofides, and A. Mingozi, "A new exact algorithm for the vehicle routing problem based on q-paths and k-shortest paths relaxation". *Annals of Operations Research* 61(1-4), 1995, pp.21-43.  
[26] Ralphs, T. K. (2003). Parallel branch and cut for capacitated vehicle routing. *Parallel Computing*, 29, 607-629.  
[27] Baldacci, R., E. Hadjiconstantinou, and A. Mingozi, "An exact algorithm for the capacitated vehicle routing problem based on a two-commodity network flow formulation". *Operations Research* 52(5), 2004, pp.723-738.  
[28] Jin, M., Liu, K., & Ekşioğlu, B. A column generation approach for the split delivery vehicle routing problem, *Operations Research Letters*, 36, 265-270, 2008.  
[29] G.B. Dantzig, J.H. Ramser, "The Truck Dispatching Problem" *Management Science*, 1959, Vol. 6, pp 81-91.  
[30] Clarke, G. and J. Wright, "Scheduling of vehicles from a central depot to a number of delivery points". *Operations Research* 12(4), 1964, pp. 568-581.  
[31] Wren, A. and A. Holliday, "Computer scheduling of vehicles from one or more depots to a number of delivery points". *Operational Research Quarterly* 23(3), 1972, pp.333-344.  
[32] Lin, S. and B. W. Kernighan, "An effective heuristic algorithm for the traveling salesman problem". *Operations Research* 21(2), 1973, pp.498-516.  
[33] Gillett, B. and L. Miller, "A heuristic algorithm for the vehicle-dispatch problem". *Operations Research* 22(2), 1974, pp. 340-349.  
[34] Foster, B. A. and D. M. Ryan, "An integer programming approach to the vehicle scheduling problem". *Operational Research Quarterly* 27(2), 1976, pp. 367-384.  
[35] Mole, R. H. and S. R. Jameson, "A sequential route-building algorithm employing a generalized savings criterion". *Operational Research Quarterly* 27(2), 1976, pp.03-511.  
[36] Christofides, N., A. Mingozi, and P. Toth, "The vehicle routing problem". In N. Christofides, A. Mingozi, P. Toth, and C. Sandi (Eds.), *Combinatorial optimization*, Chapter 11, 1979, pp. 315-338. Chichester, England: John Wiley & Sons Ltd.  
[37] Fisher, M. and R. Jaikumar, "A generalized assignment heuristic for vehicle routing". *Networks* 11(2), 1981, pp.109-124.  
[38] Beasley, J. E., "Route-first cluster-second methods for vehicle routing. *Omega* 11(4), 1983, pp. 403-408.

- [39] Altinkemer, K. and B. Gavish, "Parallel savings based heuristics for the delivery problem". *Operations Research* 39(3), 1991, pp.456–469.
- [40] Ryan, D. M., C. Hjorring, and F., "Extensions of the petal method for vehicle routing". *The Journal of the Operational Research Society* 44(3), 1993, pp.289–296.
- [41] Thompson, P. and H. Psaraftis, "Cyclic transfer algorithms for multivehicle routing and scheduling problems". *Operations Research* 41(5), 1993, pp. 935–946.
- [42] Potvin, J.-Y. and J.-M. Rousseau, "An exchange heuristic for routing problems with time windows. *Journal of the Operational Research Society* 46(12), 1995, pp.1433–1446.
- [43] Bramel, J. and D. Simchi-Levi, "A location based heuristic for general routing problems". *Operations Research* 43(4), 1995, pp.649–660.
- [44] Renaud, J., F. F. Boctor, and G. Laporte, "A fast composite heuristic for the symmetric traveling salesman problem". *INFORMS Journal on Computing* 8(2), 1996, pp.134–143.
- [45] Kindervater, G. A. P. and M. W. P. Savelsbergh, "Vehicle routing: handling edge exchanges", 1997.
- [46] Osman, I. H., "Metastrategy simulated annealing and tabu search algorithms for the vehicle routing problem. *Annals of Operations Research* 41(1-4), 1993, pp.421–451.
- [47] Taillard, E. D., "Parallel iterative search methods for vehicle routing problem. *Networks* 23(8), 1993, pp.661–673.
- [48] Gendreau, M., A. Hertz, and G. Laporte, "A tabu search heuristic for the vehicle routing problem". *Management Science* 40(10), 1994, pp. 1276–1290.
- [49] Van Breedam, A., "Improvement heuristics for the vehicle routing problem based on simulated annealing". *European Journal of Operational Research* 86(3), 1995, pp. 480–490.
- [50] Rochat, Y. and E. Taillard, "Probabilistic diversification and intensification in local search for vehicle routing". *Journal of Heuristics* 1, 1995, pp.147–167.
- [51] Xu, J. and J. Kelly, "A network flow-based tabu search heuristic for the vehicle routing problem". *Transportation Science* 30(4), 1996, pp. 379–393.
- [52] Kawamura, H., M. Yamamoto, T. Mitamura, K. Suzuki, and A. Ohuchi, "Cooperative search based on pheromone communication for vehicle routing problems". *IEICE transactions on fundamentals of electronics, communications and computer sciences* E81-A(6), 1998, pp.1089–1096.
- [53] Bullnheimer, B., R. Hartl, and C. Strauss, "An improved ant system algorithm for the vehicle routing problem". *Annals of Operations Research* 89, 1999, pp.319–328.
- [54] Toth, P. and D. Vigo (2003, Fall), 2003, "The granular tabu search and its application to the vehicle routing problem". *INFORMS Journal on Computing* 15(4), 2003, pp.333–346.
- [55] Baker, B. M. and M. A. Ayechew, "A genetic algorithm for the vehicle routing problem". *Computers & Operations Research* 30(5), 2003, pp.787–800.
- [56] Mazzeo, S. and I. Loiseau, "An ant colony algorithm for the capacitated vehicle routing". *Electronic Notes in Discrete Mathematics* 18(1), 2004, pp.181–186.
- [57] R. A. Ganiyu et al, "A Comparative Evaluation of Selected Heuristic Solutions of Vehicle Routing Problems in Supply Chain Management, *Journal of Innovative Research in Engineering and Science* 2(2), April, 2011.